Example of bianisotropic electromagnetic crystals: The spiral medium

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In this paper the electromagnetic properties of bianisotropic electromagnetic crystals are studied. The crystals are assumed to be rectangular lattices of perfectly conducting helicoidal spirals. The analytical theory of dispersion and plane-wave reflection refers to the case when the spiral step and the radius are small compared to the wavelengths in the host medium. The periods of the lattice can be arbitrary. Explicit closed-form expressions are derived for the effective material parameters of the medium in the low-frequency regime. The medium eigenmodes are elliptically polarized, and one of them propagates without interaction with the lattice. As to the other eigenmode, the lattice has strong spatial dispersion even at extremely low frequencies in the direction along the spiral axes. Numerical examples are given. An analogy between the spiral medium and the medium of loaded wires is indicated.

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I. INTRODUCTION

Electromagnetic and photonic band-gap structures attract a lot of attention in view of many potential applications (e.g., [1]). Usually, these artificial media are formed as periodic arrangements of dielectric or conducting inclusions (or voids in an isotropic matrix). The cell geometry is normally quite simple (spheres, circular cylinders, etc.). We introduce the concept of bianisotropic (or magnetoelectric) electromagnetic crystals. In these structures, as in quasihomogeneous bianisotropic media, electric and magnetic fields are coupled through the medium response [2]. In other words, electric fields cause both electric and magnetic polarizations, and also magnetic fields not only magnetize but also electrically polarize the medium. Obviously, more complicated properties of the material allow more possibilities in the design of microwave or optical devices.

The well-known optical activity phenomenon [3] was studied in composite chiral photonic crystals with a helical lattice of dielectric spheres in [4], using numerical modeling. Three-dimensional lattices (sc, fcc, and bcc) of dielectric spiral-shaped elements were considered in [5], and it was shown that the band-gap structure depends on the geometry of the elements, but not only on the lattice geometry. Microwave magnetoelectric coupling in media can be due to nonreciprocal properties of inclusions [3,6] or to the complicated geometrical structure of the medium [2]. In this work we explore the second possibility and study a *spiral medium*, a periodic medium formed by long spiral ideally conducting inclusions (helixes). The effective-medium regime of this medium was considered in [7].

The special interest of the structure under consideration is based on its wide range of possible applications, beginning with the design of frequency and polarization filters and ending with the synthesis of high impedance surfaces in the microwave frequency region [8]. To simplify the study (without loss of general properties), we model helices as sets of connections of straight wires and coils, as depicted in Fig. 1, in the same manner as was done in [9]. The same structure was also considered in [10] and its plasmonic behavior was revealed.

In this paper we present an analytical model of a two-dimensional lattice of infinitely long and thin parallel perfectly conducting helices. In this the structure the polarizations of the eigenwaves in the medium become elliptical due to bianisotropy effects. At the same time, these magnetoelectric coupling effects are combined with spatial dispersion effects as in electromagnetic (or photonic) crystals where the spatial resonances of the lattice determine the stop bands. Moreover, the helicoidal spirals have special resonant properties (the parallel resonance of the loop inductance and the interturn capacitance) which lead to the resonant behavior of the whole medium at frequencies close to the helix individual resonance (which is the antiresonance). The structure under consideration can potentially offer great opportunities for control of the dispersion properties of artificial materials, and it can possibly be used for prospective frequency and polarization filtering of the microwave signals.

II. ANALYTICAL APPROACH

Let the spiral medium be formed by a rectangular lattice of helicoidal spirals with periods $a\times b$, spiral pitch c, radius of a turn r, and radius of wires r_0 (see Fig. 1). In this theory

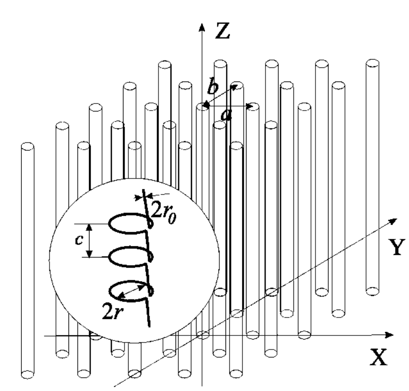

FIG. 1. Geometry of the spiral medium.

we restrict consideration to the case when the wavelength in the host medium and the lattice periods are large compared to the spiral pitch and diameter: $r,c \ll a,b,\lambda$. The periods of the lattice can be arbitrary with respect to λ. Also, assume that the radius of the wires is small compared with the spiral pitch and turn diameter: $r_0 \ll 2r,c$.

We consider the general case of wave propagation (arbitrary angles between the wave vector and the spiral axes). To take into account the electromagnetic interaction of the helices, we model every helix as lines of electric and magnetic currents both referred to the spiral axis. This means that we neglect the effect of helix polarization in the direction orthogonal to its axis. An electric current line produces axial electric field and does not produce any axial magnetic field, and a magnetic current line produces axial magnetic field and does not produce any axial electric field. This means that there is no electromagnetic interaction between the electric and magnetic currents. Therefore, the interaction of helices can be described through known interaction constants obtained for lattices from straight line currents in [11,12]. However, spirals possess bianisotropic properties, which means that the local electric field excites magnetic currents in spirals, and the local magnetic field excites electric currents. To describe this effect we use the model of a helix suggested in [9], which replace the uniform helix by a set of collinear straight wires connected to the split loops as shown in Fig. 1. The analytical theory [9] gives a simple expression for the current induced in such a spiral by local electric and magnetic fields. Using this simple model allows one to derive an explicit scalar dispersion equation, as was done for a lattice of straight conducting wires in [11].

A. Polarization of an individual spiral

We assume the radius r and the pitch c to be small enough compared to the wavelength and the lattice periods so that the current distribution in the turns of any spiral can be considered as uniform. Consider an individual helix excited by external electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields. Assume that the dependence of $\mathbf{E}$ and $\mathbf{H}$ on the helix axis z is harmonic: $e^{-jq_z z}$. A unit length piece of the spiral can be modeled as a circuit with impedance Z relating the electromotive force $\mathcal{E}$ and the current I at its center [9]:

$$I = \mathcal{E}/Z. \tag{1}$$

The electromotive force per unit length is the sum of the axial component of the electric field and the following contribution of the magnetic field:

$$\mathcal{E} = E_z - \eta\gamma H_z, \quad \gamma = \pm jk\pi r^2/c. \tag{2}$$

The signs + or − in formula (2) correspond to the right or left handedness of the spiral. From Eq. (2) we can see that an elliptically polarized wave with $E_z - \eta\gamma H_z = 0$ does not interact with the spiral.

The impedance Z is the sum of the impedances of straight wire pieces with unit total length and the impedances of loops per unit length of the spiral:

$$Z = Z_{\text{wire}} + Z_{\text{loops}}. \tag{3}$$

An expression for Z_{wire} was derived in [11]:

$$Z_{\text{wire}} = \frac{\eta(k^2 - q_z^2)}{4k}\left(1 - j\frac{2}{\pi}\left\{\ln\frac{\sqrt{k^2 - q_z^2}\,r_0}{2} + \gamma_e\right\}\right), \tag{4}$$

where η is the free-space wave impedance, $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the wave number in free space, and ε_0 and μ_0 are the permittivity and permeability of free space, $\gamma_e \approx 0.5772$ is the Euler constant. It is easy to modify η and k for the case when the host medium is a homogeneous isotropic dielectric instead of free space.

The impedance of the loops Z_{loops} was calculated in [9] within the frame of the quasistatic model as a parallel connection of input capacitance and inductance, both referred to the loop split. The input inductance is the sum of the self-inductance of the loop and the mutual inductance between the reference loop and all other loops of the same spiral. The input capacitance includes the self-capacitance of the loop and the mutual capacitance between the reference loop and other loops of the spiral.

We can ignore the real part of the loop impedance (radiation resistance of the loop). In regular structures of arbitrary scatterers the radiation resistance of an individual scatterer and the real part of the interaction constant of the lattice from those scatterers give the imaginary part of the dispersion equation. If the scatterers are lossless the imaginary part of the dispersion equation is zero [6,11,12]. So the radiation resistance per unit length of the spiral cancels the real part of the interaction constant. For the imaginary part of Z_{loops} we have

$$\operatorname{Im}\{Z_{\text{loops}}\} = \frac{k^2 - q_z^2}{ck^2}\,\frac{\omega(L_0 + M)}{1 - \omega^2(L_0 + M)C_0}, \tag{5}$$

$$L_0 = \mu_0 r\left(\ln\frac{8r}{r_0} - 2\right), \tag{6}$$

$$M = \frac{\mu_0 r}{2\pi}\sum_{n=1}^{+\infty} G\left(\frac{2r}{cn}\right), \tag{7}$$

$$C_0 = \frac{4\pi^2\epsilon_0 r}{\ln\frac{8r}{r_0} - \kappa K}. \tag{8}$$

The factor $(k^2 - q_z^2)/k^2$ in Eq. (5) takes into account the z dependence of the spiral current $I(z) = I(0)e^{-jq_z z}$. $G(x)$ is a tabulated function [13], for which a good approximation for the case $a \leq 4r$ is known [13]:

$$G(x) = \frac{\pi^2 x^3}{4}\left(1 - \frac{3}{4}x^2 + \frac{75}{128}x^4 + \cdots\right). \tag{9}$$

κ and K determine the elliptic integral:

$$K=\int_0^{\pi/2}\frac{dx}{\sqrt{1-\kappa^2\sin^2 x}}, \quad \kappa=\frac{r^2}{r^2+0.25c^2}. \tag{10}$$

Thus, using Eq. (2) together with Eqs. (4) and (5) we obtain a model of the individual spiral polarization by the local electromagnetic field.

B. Dispersion equation

The dispersion equation can be derived following [11,12], where the case of a wire medium was considered. Assume the distribution of spiral currents in a lattice to be related to the wave vector $\mathbf{q}=(q_x,q_y,q_z)^T$ as

$$I_{m,n}(z)=Ie^{-j(q_x am+q_y bn+q_z z)}, \tag{11}$$

where I is the current in the reference spiral. An infinite lattice of spirals with currents (11) produces the following local electric and magnetic fields acting on the reference spiral:

$$E_z=C(k,q_x,q_y,q_z)I, \tag{12}$$

$$H_z=C(k,q_x,q_y,q_z)I_m/\eta^2, \quad I_m=\eta\gamma I, \tag{13}$$

where I_m is the magnetic current in the reference spiral, and $C(k,q_x,q_y,q_z)$ is the so-called interaction factor of the wire lattice calculated in [11]:

$$C(k,q_x,q_y,q_z)=-j\frac{\eta(k^2-q_z^2)}{2kb}\left[\frac{1}{k_x^{(0)}}\frac{\sin k_x^{(0)}a}{\cos k_x^{(0)}a-\cos q_x a}+\sum_{n\neq 0}\left(\frac{1}{k_x^{(n)}}\frac{\sin k_x^{(n)}a}{\cos k_x^{(n)}a-\cos q_x a}-\frac{b}{2\pi|n|}\right)+\frac{b}{\pi}\left(\ln\frac{\sqrt{k^2-q_z^2}b}{4\pi}+\gamma\right)+j\frac{b}{2}\right], \tag{14}$$

$$k_x^{(n)}=-j\sqrt{\left(q_y+\frac{2\pi n}{b}\right)^2+q_z^2-k^2}, \tag{15}$$

where we choose $\mathrm{Re}\{\sqrt{()}\}>0$.

Thus, the following dispersion equation can be written using Eq. (2) with Eqs. (12) and (13):

$$\frac{Z}{1-\gamma^2}=C(k,q_x,q_y,q_z). \tag{16}$$

As was noticed above, in this equation the imaginary parts cancel out. From Eqs. (14) and (4) it follows that

$$\mathrm{Re}\{Z_{\mathrm{wire}}\}=\mathrm{Re}\{C\}=\frac{\eta(k^2-q_z^2)}{4k}. \tag{17}$$

The rule of the cancellation of the loop radiation resistance gives, by the way, an expression for it (this resistance remained unknown in the theory [9]):

$$\mathrm{Re}\{Z_{\mathrm{loops}}\}=-\gamma^2\frac{\eta(k^2-q_z^2)}{4k}. \tag{18}$$

Finally, we obtain the real valued equation

$$\frac{1}{1+|\gamma|^2}\left[\ln\frac{b}{2\pi r_0}-|\gamma|^2\left(\ln\frac{\sqrt{k^2-q_z^2}r_0}{2}+\gamma_e\right)\right]+\frac{2\pi k}{\eta(k^2-q_z^2)}\frac{\mathrm{Im}\{Z_{\mathrm{loops}}\}}{1+|\gamma|^2}+\frac{\pi}{bk_x^{(0)}}\frac{\sin k_x^{(0)}a}{\cos k_x^{(0)}a-\cos q_x a}+\sum_{n\neq 0}\left(\frac{\pi}{bk_x^{(n)}}\frac{\sin k_x^{(n)}a}{\cos k_x^{(n)}a-\cos q_x a}-\frac{1}{2|n|}\right)=0. \tag{19}$$

The series in Eq. (19) has good convergence (as $1/n^3$), so, it is easy to solve and analyze this equation numerically. The dispersion equation (19) describes only the extraordinary wave which interacts with spirals and excites them. The ordinary wave satisfies $E_z-\eta\gamma H_z=0$ and travels in the host medium without interaction with the spirals.

C. Reflection coefficient from a half space

The dispersion characteristics of the lattice determine the reflection coefficient from an interface between free space and a half space filled with this lattice [6,11,12,14]. The expression for the reflection coefficient in the case when an incident wave with wave vector $\mathbf{k}=(k_x,k_y,k_z)^T$ excites the only extraordinary wave in the lattice (this polarization must be elliptic and orthogonal to the elliptic polarization of the incident wave, exciting only the ordinary wave $E_z-\eta\gamma H_z=0$) is as follows:

$$R=-e^{-jk_x a}\prod_{n=1}^{+\infty}e^{jk_x a}\frac{\sin[(q_x^n-k_x)a/2]}{\sin[(q_x^n+k_x)a/2]}, \tag{20}$$

where q_x^n are all possible solutions of the dispersion equation (19) with $q_y=k_y$, $q_z=k_z$. The incident wave that excites only the ordinary mode has zero reflection coefficient and total transmission if the host medium is free space.

III. MATERIAL PARAMETERS

In the low-frequency regime we can model the spiral medium as a uniaxial chiral medium described by bianisotropic material equations

$$\mathbf{D}=\varepsilon_0\bar{\bar{\varepsilon}}\cdot\mathbf{E}+j\sqrt{\varepsilon_0\mu_0}\bar{\bar{\kappa}}\cdot\mathbf{H},$$

$$\mathbf{B}=\mu_0\bar{\bar{\mu}}\cdot\mathbf{H}-j\sqrt{\varepsilon_0\mu_0}\bar{\bar{\kappa}}\cdot\mathbf{E}.$$

Here, the material parameters are dyadics of the following form:

$$\bar{\bar{\varepsilon}} = \varepsilon \mathbf{z}_0\mathbf{z}_0 + \mathbf{x}_0\mathbf{x}_0 + \mathbf{y}_0\mathbf{y}_0,$$

$$\bar{\bar{\mu}} = \mu \mathbf{z}_0\mathbf{z}_0 + \mathbf{x}_0\mathbf{x}_0 + \mathbf{y}_0\mathbf{y}_0, \quad \bar{\bar{\kappa}} = \kappa \mathbf{z}_0\mathbf{z}_0.$$

Such a medium possesses two eigenmodes for every direction of propagation with so-called *axial impedance* introduced in [2]

$$Z_z^{\pm} = \frac{E_{z_\pm}}{H_{z_\pm}} = \frac{\eta}{2j\kappa}[(\varepsilon-\mu) \pm \sqrt{(\varepsilon-\mu)^2+4\kappa^2}] \tag{21}$$

and the propagation factors [2]

$$q_\pm^2 = \frac{2k^2(\varepsilon\mu-\kappa^2)}{2(\varepsilon\mu-\kappa^2)q_z^2/q^2+[(\varepsilon+\mu)\mp\sqrt{(\varepsilon-\mu)^2+4\kappa^2}](q^2-q_z^2)/q^2}. \tag{22}$$

The ordinary mode, which does not interact with the spiral medium, has the axial impedance $Z_z^- = \eta\gamma$ and $q_- = k$. The absolute value of the extraordinary mode propagation factor $q_+ = q$ can be found by solving the dispersion equation (19) in the quasistatic approximation, i.e., using the Taylor expansion of trigonometric functions as was done in [11,12]:

$$q^2 = k^2 - k_0^2,$$

$$k_0^2 = \frac{2\pi}{ab}\left\{\frac{1}{1+|\gamma|^2}\left[\ln\frac{b}{2\pi r_0} + \frac{2\pi}{\eta k c}\,\frac{\omega(L_0+M)}{1-\omega^2(L_0+M)C_0} - |\gamma|^2\left(\ln\frac{\sqrt{k^2-q_z^2}\,r_0}{2}+\gamma_e\right)\right] + F\left(\frac{a}{b}\right)\right\}^{-1}, \tag{23}$$

where

$$F(x) = \sum_{n=1}^{+\infty}\left(\frac{\coth(\pi n x)-1}{n}\right) + \frac{\pi x}{6}.$$

For a square grid $F(1) = 0.5275$.

Solving the system of three equations $Z_- = \eta\gamma$, $q_- = k$, and $q_+ = q$ together with Eqs. (21), (22), and (23) for three unknowns ε, μ, and κ, we obtain explicit expressions for the material parameters depending on the frequency and on the propagation factor:

$$\varepsilon = 1 - \frac{1}{1+|\gamma|^2}\,\frac{k_0^2}{k^2-q_z^2}, \tag{24}$$

$$\mu = 1 - \frac{|\gamma|^2}{1+|\gamma|^2}\,\frac{k_0^2}{k^2-q_z^2}, \tag{25}$$

$$\kappa = \frac{\mathrm{Im}\{\gamma\}}{1+|\gamma|^2}\,\frac{k_0^2}{k^2-q_z^2}. \tag{26}$$

The lattice of spirals is a nonconventional uniaxial chiral medium. Even at extremely low frequencies it possesses strong spatial dispersion in the direction along the spiral axes. The nature of this phenomenon is the same as for wire media [15]. It results from the infinite length of spirals in the axial direction.

IV. NUMERICAL EXAMPLES

We have studied numerically the dispersion and reflection properties of spiral media with various geometrical parameters. The main principles and properties can be illustrated with an example of a square lattice $a\times a$ from infinite right-handed spirals with turn radius $r=0.15a$, spiral pitch $c=0.5a$, and wire radius $r_0=0.01a$. The dispersion plot for this medium is presented in Fig. 2 (thick lines), where $\Gamma=(0,0,0)^T$, $X=(\pi/a,0,0)^T$, and $M=(\pi/a,\pi/a,0)^T$ are points in the first Brillouin zone. The thin lines represent the dispersion curves for free space to show the difference between our medium and free space.

We calculated the reflection coefficient R for the case of normal incidence on a half space filled by this spiral medium. The incident wave polarization is assumed to be exciting the only extraordinary wave in the lattice. The frequency range corresponds to the regime of the single propagating mode ($ka<2\pi$). In Fig. 3 the reflection coefficient R is

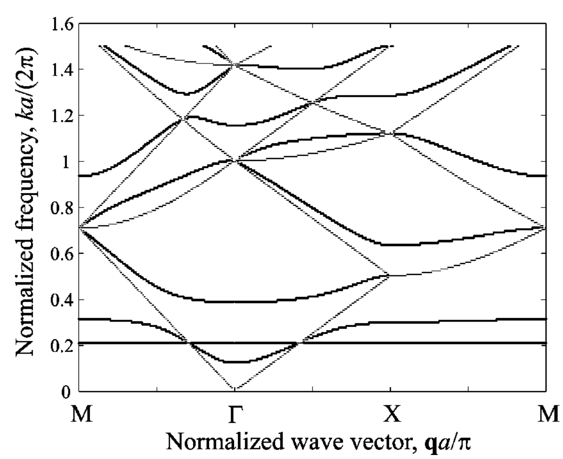

FIG. 2. Dispersion plot for a spiral medium formed by a square grid $a\times a$ of infinite spirals with turn radius $0.15a$, pitch $0.5a$, and wire radius $0.01a$.

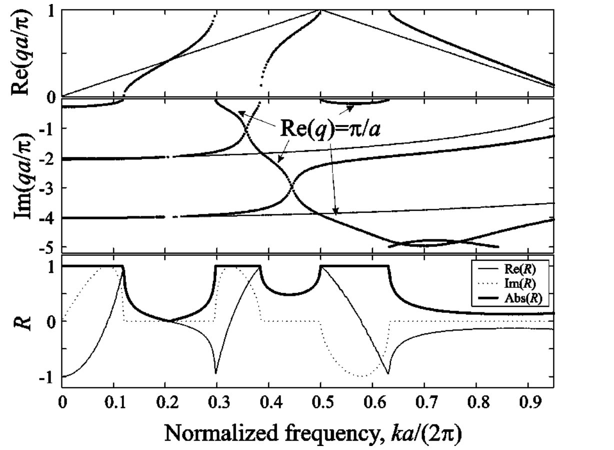

FIG. 3. Reflection coefficient from a half space filled with the same spiral medium (normal incidence, spiral axes are parallel to the interface).

shown as a function of the normalized frequency $ka/(2\pi)$ together with the corresponding propagation factors plotted above it. The modes are of two types: propagating $\mathrm{Im}(q)=0$ and decaying $\mathrm{Im}(q)<0$. The decaying modes can be further split into two types: conventional modes with exponential decay $[\mathrm{Re}(q)=0, \mathrm{Im}(q)<0]$ and nonconventional (complex) modes, which can be considered as exponentially decaying with alternating directions of the induced currents in the spirals along the wave propagation direction $[\mathrm{Re}(q)=\pi/a, \mathrm{Im}(q)<0]$. In Fig. 3 at the top the real parts of q for the propagating modes are plotted, in the central part of Fig. 3 the imaginary parts of q for decaying modes are plotted, and the decaying modes of the second type are marked with the legend $\mathrm{Re}(q)=\pi/a$. In the plots for $\mathrm{Re}(qa/\pi)$ and $\mathrm{Im}(qa/\pi)$, thin lines show the modes for free space considered as a lattice of spirals with zero polarizability.

For the same lattice we have calculated the material parameters for the case when the propagation direction is orthogonal to the spiral axes $q_z=0$. The results are plotted in Fig. 4 as functions of the normalized frequency $ka/2\pi$. We

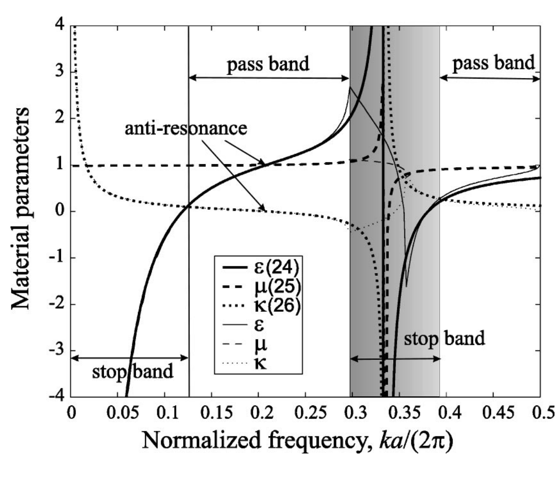

FIG. 4. Effective material parameters for the same spiral medium ($q_z=0$) calculated using the approximate expressions (24), (25), and (26) (thick lines) and the exact solution (thin lines).

have compared the approximate expressions (24), (25), and (26) with the exact solution obtained using the dispersion equation (19) and the same homogenization method based on finding the mode axial impedances (21) and propagation factors (22). We have found that the approximation is accurate for $ka/(2\pi)<0.27$. The homogenization of our particular spiral medium is possible only for $ka/(2\pi)<0.3$. Indeed, in the region $0.3<ka/(2\pi)<0.39$ the medium has no propagating modes and two main decaying modes (with decay factors of the same order) (see Fig. 3). One of the modes has $\mathrm{Re}(q)=\pi/a$, as if the effective wavelength in the medium were equal to the double lattice period. Such modes cannot exist in a homogeneous medium. Thus, we can conclude that for electric fields in the z direction strong spatial dispersion effects exist at all frequencies, whereas for the orthogonal directions of the electric field spatial dispersion can be observed near the resonance of the spirals.

The analytical and numerical results can be summarized as follows.

(1) The eigenmodes have an elliptic polarization. For the ordinary mode with the eigenpolarization that satisfies $E_z-\eta\gamma H_z=0$, the medium is transparent in the whole frequency range where our model is valid.

(2) There exists a low-frequency band gap for the extraordinary mode. At its upper edge $[ka/(2\pi)=0.11$ in our example$]$ the lattice behaves as a magnetic wall for the plane-wave reflection $(R=+1)$. Notice that a similar result was obtained earlier for lattices of inductively loaded wires [12].

(3) At the frequency of the parallel resonance of spirals $[ka/(2\pi)=0.2$ in our example$]$ the medium becomes transparent for the extraordinary mode, too. Notice that a similar result was obtained earlier for lattices from wires loaded with parallel LC circuits [12].

(4) For different ratios of the turn radius and the pitch the second miniband for the extraordinary mode appears either in the first $(ka<\pi)$ or in the second $(\pi<ka<2\pi)$ frequency band. By tuning this ratio, one can obtain a very narrow passband (e.g., the ratio of the central frequency to the bandwidth can be made of the order of 10^3).

(5) Low-frequency material parameters have been found and analyzed for the case of normal propagation with respect to the spiral axes. The value of the magnetic permeability μ is found to be always positive and close to 1. At low frequencies, the dielectric permittivity ϵ is negative as for wire media [11]. The chirality factor κ has nonzero and positive values (right-handed spirals) at low frequencies.

V. CONCLUSION

The electromagnetic properties of a bianisotropic electromagnetic crystal (spiral medium) have been studied analytically. The transcendental dispersion equation was obtained in the closed form and numerically solved. The dispersion curves and the reflection coefficient were calculated. Due to the bianisotropy, the eigenmodes of the crystal have elliptical polarization (with the axial ratio directly related to the spiral winding parameters). The chirality factor of the homogenized medium is very high at low frequencies, as well as the medium permittivity. A low-frequency band gap is obtained, which is inherent for lattices of wires. At the frequency that corresponds to the upper edge of this gap, the interface of the half space filled by this medium behaves as a magnetic wall for the eigenmode interacting with the lattice (extraordinary wave). This frequency is also the lower edge of the miniband related to the antiresonance of the individual helix. Within this miniband there is a frequency at which the lattice medium is completely transparent in all directions (the degenerate case when both modes coincide). At frequencies higher than the antiresonance one, a stop band appears with strong spatial dispersion in the directions perpendicular to the spiral axes.

The dispersion plots for the spiral media are similar to those of a lattice of wires loaded by resonant parallel LC circuits. In the low-frequency range, simple analytical formulas were defined for the effective bianisotropic uniaxial material parameters depending on the frequency and the wave vector. It was shown that strong low-frequency dispersion is inherent to this medium. This effect results from the infinite extent of spirals in the axial direction.

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